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Two-sided non-collapsing curvature flows

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Abstract. It was recently shown that embedded solutions of curvature flows in Euclidean space with concave (convex), degree one homogeneous speeds are interior (exterior) non-collapsing [6]. These results were subsequently extended to hypersurface flows in the sphere and hyperbolic space [11]. In the first part of the paper, we show that locally convex solutions are exterior non-collapsing for a larger class of speed functions than previously considered; more precisely, we show that the previous results hold when convexity of the speed function is relaxed to inverse-concavity. We note that inverse-concavity is satisfied by a large class of concave speed functions [4]. Thus, as a consequence, we obtain a large class of two-sided non-collapsing flows, whereas previously two-sided non-collapsing was only known for the mean curvature flow. In Section 3, we demonstrate the utility of two sided non-collapsing with a straightforward proof of convergence of compact, convex hypersurfaces to round points. The proof of the non-collapsing estimate is similar to those of the previous results mentioned, in that we show that the exterior ball curvature is a viscosity supersolution of the linearised flow equation. The new ingredient is the following observation: Since the function which provides an upper support in the derivation of the viscosity inequality is defined on $M \times M$ (or TM in the 'boundary case'), whereas the exterior ball curvature and the linearised flow equation depend only on the first factor, we are privileged with a freedom of choice in which second derivatives from the extra directions to include in the calculation. The optimal choice is closely related to the class of inverse-concave speed functions.

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